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Phase transitions in transmission lines with long-range fluctuating correlated disorder

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ABSTRACT

In this work we study the localization properties of the disordered classical dual transmission lines, when the values of capacitances $\{C_j\}$ and inductances $\{L_j\}$ fluctuate in phase in the form $C_j = C_0 + b\,\sin(2\pi x_j)$ and $L_j = L_0 + b\,\sin(2\pi x_j)$, where b is the fluctuation amplitude. $\{x_j\}$ is a disordered long-range correlated sequence obtained using the Fourier filtering method which depends on the correlation exponent α. To obtain the transition point in the thermodynamic limit, we study the critical behavior of the participation number D. To do so, we calculate the linear relationship between $\ln(D)$ versus $\ln(N)$, the relative fluctuation η_D and the Binder cumulant B_D. The critical value obtained with these three methods is totally coincident between them. In addition, we calculate the critical behavior of the normalized localization length $\Lambda(b)$ as a function of the system size N. With these data we build the phase diagram (b,α), which separates the extended states from the localized states. A final result of our work is the disappearance of the conduction bands when we introduce a finite number of impurities in random sites. This process can serve as a mechanism of secure communication, since a little alteration of the original sequence of capacitances and inductances, can destroy the band of extended states.

1. Introduction

The study of low-dimensional disordered systems with long-range and short-range correlations have attracted scientific interest in the last decades, because this type of disordered systems can support extended states or a transition from localized states to extended states [1–14]. However, for uncorrelated disordered systems, the Anderson localization theory predicts the absence of extended states for low-dimensional systems [15–17]. A very interesting review on Anderson localization in low-dimensional systems with correlated disorder, can be found in the work of Izrailev et al. [18].

On the other hands, due to the analogy of the transmission lines to quantum (electrons and excitons) and classical (atomic vibrations) systems, recently, analytical and numerical studies of disordered electrical transmission lines (TL) have been proposed [19–22]. In Refs. [20,21], the localized-extended transition was studied considering a ternary map to distribute the disorder in the capacitances (diagonal disorder) of the TL. The phase diagram separating localized states from extended states was obtained using the finite-size scaling method. Given that the long-range correlated sequences were generated using two very different methods: the Fourier filtering method [23] and the Ornstein–Uhlenbeck process [24], respectively, the phase diagrams are very different. In addition, the behavior of the Rényi entropies [25] of TL with Fibonacci distribution of two values of inductances have been recently studied [22]. In this case, the diagonal and off-diagonal terms of the dynamic equation vary simultaneously. The electric current function $I(\omega)$ of the TL found in this paper can be classified as extended, intermediate and localized, in complete accordance with the behavior of the quantum wave function of Fibonacci one-dimensional tight-binding systems.

In the present work we study the localization behavior of the classical dual electrical TL with long-range correlated disorder in the distribution of capacitances $\{C_j\}$ and inductances $\{L_j\}$. We introduce the disorder from a random sequence with a power spectrum $S(k) \propto k^{-(2\alpha-1)}$, where $\alpha \geq 0.5$ is the correlation exponent. From the resulting long-range correlated sequence $\{x_j\}$ we generate the disordered distribution of capacitances and inductances in the following form $C_j = C_0 + b\,\sin(2\pi x_j)$ and $L_j = L_0 + b\,\sin(2\pi x_j)$. In this way, the capacitances C_j and inductances L_j vary in phase around C_0 and L_0, respectively. The parameter b measures the amplitude of the fluctuation and varies from $b = 0$ (periodic case) to $b < \min(C_0, L_0)$, to avoid a negative value of capacitances and inductances.

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If the correlation exponent α take the value $\alpha=0.5$, we obtain a random sequence (white noise) and the TL is in the localized state for every frequency ω. For $\alpha>0.5$ and depending on the values of the fluctuation parameter b, the electric current function $I(\omega)$ can be a localized function or an extended function, for fixed system size N. This implies the existence of a critical value of the correlation exponent $\alpha_c(b)$ for each value b of the fluctuation amplitude. To determine the localization properties of the electric current $I(\omega,b,\alpha,N)$ in the thermodynamic limit, we study the critical behavior of the participation number $D(\omega,b,\alpha,N)$ using three different methods: (a) study of the linear relationship between $\ln(D)$ and $\ln(N)$, (b) study of the relative fluctuation $\eta_D(\omega,b,\alpha,N)$ of the participation number D as a function of the system size N and (c) study of the Binder cumulant $B_D(\omega,b,\alpha,N)$ of the participation number D as a function of the system size N. In addition, we study the behavior of the normalized localization length $\Lambda(b)$ as a function of the system size N to determine the critical curve $b_c(\alpha)$. With the obtained results we build the phase diagram in the plane (b,α).

The problem studied in this paper is similar to the problem studied by Shima et al. [11] in the electronic case. They studied the localization properties of electron eigenstates in one-dimensional systems with long-range correlated diagonal disorder. The phase diagram they get is similar to the phase diagram obtained in our work.

This paper is organized as follows. Section 2 describes the model and methods used to calculate the critical behavior of some quantities. In addition, we indicate the numerical procedure to calculate them. Section 3 presents the numerical results for the set of quantities under study as a function of the correlation exponent α, the amplitude of the fluctuation b and the system size N. The critical behavior in the plane (b,α) is presented. In Section 4 we discuss the secure communication. Finally, the conclusions of our work are presented in Section 5.

2. Model and method

We consider the classical electrical dual TL with horizontal capacitances C_j and vertical inductances L_j. Application of Kirchhoff's Loop Rule to three successive unit cells of the circuit leads to the following linear relation between the currents circulating in the $(j-1)$-th, j-th and $(j+1)$-th cells [19–22]

$$d_jI_j-L_{j-1}I_{j-1}-L_jI_{j+1}=0 \quad (1)$$

where $d_j=(L_{j-1}+L_j-(1/\omega^2C_j))$ and ω are the frequencies. In this work we introduce the fluctuating correlated disorder in C_j and L_j in the following form:

$$C_j=C_0+b\sin(2\pi x_j)$$
$$L_j=L_0+b\sin(2\pi x_j) \quad (2)$$

where parameter b measures the amplitude of each fluctuation. In the last expression, $\{x_j\}$ is a long-range correlated sequence generated using the Fourier filtering method [23]. The long-range correlation of the sequence $\{x_j\}$ is controlled by the correlation exponent α, where $\alpha\geq 0.5$. The case $\alpha=0.5$ corresponds to a random sequence (white noise). On the other hand, we regain the periodic TL when $b=0$. The specific values of each long-range correlated sequences $\{C_j\}$ and $\{L_j\}$ are determined by two parameters: the correlation exponent α and the fluctuation amplitude b.

Using the following definition:

$$\gamma_j\equiv L_j\left(\frac{I_{j+1}}{I_j}\right) \quad (3)$$

Eq. (1) can be written as a recurrence equation

$$\gamma_j=\left(L_{j-1}+L_j-\frac{1}{\omega^2C_j}\right)-\frac{L_{j-1}^2}{\gamma_{j-1}} \quad (4)$$

To study the localization behavior of the disordered dual transmission lines, we use a set of quantities that depends on the frequency ω, the parameters b and α, and the system size N. We calculate the electric current function $I(\omega,b,\alpha,N)$, the localization length $\xi(\omega,b,\alpha,N)$, the Shannon entropy $S(\omega,b,\alpha,N)$, the participation number $D(\omega,b,\alpha,N)$ and the inverse participation ratio $IPR(\omega,b,\alpha,N)$, where $IPR=D^{-1}$. All quantities are calculated using the normalized electric current function $I_j:\sum_{j=1}^N |I_j|^2=\sum_{j=1}^N Q_j=1$, with $Q_j=|I_j|^2$ and $0\leq Q_j\leq 1$.

The localization length $\xi(\omega,b,\alpha)$ is defined as

$$\xi^{-1}(\omega,b,\alpha)=\lim_{N\to\infty}\frac{1}{N}\sum_{j=1}^N \ln\left|\frac{I_{j+1}}{I_j}\right| \quad (5)$$

where I_j and I_{j+1} are the currents circulating in cells j and $(j+1)$, respectively. Using the γ_j (3), $\xi(\omega,b,\alpha)$ can be written in the following form:

$$\xi^{-1}(\omega,b,\alpha)=\lim_{N\to\infty}\frac{1}{N}\sum_{j=1}^N \ln\left|\frac{\gamma_j}{L_j}\right| \quad (6)$$

For finite system size N we can define the normalized localization length in the following way: $\Lambda(\omega,b,\alpha,N)=\xi(\omega,b,\alpha,N)/N$. With this quantity and for fixed parameters b and α, and fixed system size N, we can differentiate between localized and extended states according to the following criteria: if $\Lambda(\omega,b,\alpha,N)\geq 1$, the electric current function $I(\omega)$ is an extended function, and if $\Lambda(\omega,b,\alpha,N)<1$, $I(\omega)$ is a localized function.

The Shannon entropy is defined as

$$S(\omega)=-\sum_{j=1}^N Q_j\ln Q_j \quad (7)$$

and the participation number $D(\omega)$ is defined as

$$D^{-1}(\omega)=IPR=\sum_{j=1}^N Q_j^2 \quad (8)$$

where IPR is the inverse participation ratio. All these quantities are bounded in the following form:

$$0\leq S\leq\ln(N),\quad 1\leq D\leq N,\quad \frac{1}{N}\leq IPR\leq 1 \quad (9)$$

Sometimes it is useful to work with the normalized inverse participation ratio $NIPR$ defined as $NIPR=(N\times IPR)$ which is also a bounded quantity, namely, $1\leq NIPR\leq N$. For the periodic case, we have $NIPR_0=1.5$.

For fixed frequency ω and fixed system size N, we will study the behavior of these quantities: $\Lambda(b,\alpha)$, $D(b,\alpha)$, $S(b,\alpha)$ and $IPR(b,\alpha)$ as a function of the fluctuation amplitude b and the correlation exponent α. We will compare the behavior of these quantities in the extended states and in the localized states. In this way we can characterize each quantity for each kind of state. Given that every quantity depends on the choice of the disordered sequence $\{C_j\}$ and $\{L_j\}$, we take the mean over 10^4 different configurations to obtain a typical value of every quantity. Consequently, in what follows, all figures show only averaged quantities, except for the spatial dependence of the electric current function $I(b,\alpha)$ (see Fig. 9 below).

To determine the localization properties of the electric current function $I(\omega,b,\alpha,N)$ of the disordered transmission line in the thermodynamic limit, we will study the critical behavior of two quantities: (i) the participation number $D(\omega,b,\alpha,N)$, and (ii) the normalized localization length $\Lambda(\omega,b,\alpha,N)$.

(i) The critical behavior of the participation number D.

The participation number D will be studied using the following three different methods: (a) study of the linear relationship between $\ln(D)$ and $\ln(N)$, (b) study of the relative fluctuation $\eta_D(\omega,b,\alpha,N)$ of the participation number D as a function of the system size N and (c) study of the Binder cumulant $B_D(\omega,b,\alpha,N)$ of the participation number D as a function of the system size N.

(a) Linear relationship between $\ln(D)$ and $\ln(N)$.

For extended states, the participation number $D(\omega,b,\alpha,N)$ diverges proportional to the number of sites N, but remains finite for localized states [26]. Therefore, when the slope m of the linear relationship between $\ln(D)$ and $\ln(N)$ goes to $m=1.0$, we obtain the critical value $\alpha_c=\alpha_c(b)$ of the correlation exponent α, because for $\alpha \geq \alpha_c$, we have $m(\alpha \geq \alpha_c)=1.0$. As a consequence, for $\alpha \geq \alpha_c$, all states are extended states.

(b) The relative fluctuation $\eta_D(\omega,b,\alpha,N)$ of the participation number D, is defined as

$$\eta_D(\omega,b,\alpha,N)=\frac{\Delta D}{\langle D\rangle}=\frac{\sqrt{\langle D^2\rangle-\langle D\rangle^2}}{\langle D\rangle} \quad (10)$$

where $\Delta D=\sqrt{\langle D^2\rangle-\langle D\rangle^2}$ is the fluctuation of the participation number, and $\langle D^2\rangle$ is the averaged squared participation number. For long-range disordered systems, it has been shown that the distribution function of the participation number $D(\omega,b,\alpha,N)$ is scale invariant at the Anderson transition [27,26]. This scale invariance has been used to study the transition from localized to extended states in different models [26,28–31]. The relative fluctuation $\eta_D(\omega,b,\alpha,N)$ behaves in the following form: for extended states, η_D goes to zero for increasing system size N, and for localized states, η_D grows with increasing system size N converging to a finite value. In consequence, when the system size goes to infinity, $N\to\infty$, the relative fluctuation η_D tends to a step function. In this limiting case, a discontinuity appears which separates the extended states from the localized states. Therefore, when we study the behavior of η_D as a function of the critical parameter α, for fixed fluctuation amplitude b, the curves with different system sizes N will cross at a single point. This point indicates the value of the critical parameter $\alpha_c(b)$ for the transition from localized state to extended state.

(c) The Binder cumulant $B_D(\omega,b,\alpha)$ of the participation number D is defined as

$$B_D(\omega,b,\alpha)=1-\frac{\langle D^4\rangle}{3\langle D^2\rangle^2} \quad (11)$$

where $\langle D^4\rangle$ is the average of the participation number to the fourth power. From statistical physics it is known that the Binder cumulant [32,33] of the order parameter permits the location of the critical parameter. In the thermodynamic limit, where the system size $N\to\infty$, the Binder cumulant function jumps abruptly from a constant value ($B_D\to\frac{2}{3}\approx 0.67$, for extended states) to zero ($B_D\to 0$, for localized states) at the critical parameter $\alpha_c(b)$. For finite system size N, the Binder cumulant function rounds and changes smoothly to zero. As a consequence, the Binder cumulants for different system sizes N cross at the critical point $\alpha_c=\alpha_c(b)$, locating in this way the position of the transition from localized states to extended states. In summary, we will use three different methods to obtain the critical value $\alpha_c=\alpha_c(b)$, corresponding to the transition from localized states to extended states in the disordered long-range correlated dual transmission line.

(ii) Critical behavior of the normalized localization length Λ.

We will calculate the normalized localization length $\Lambda(b,\alpha,N)$ for different system sizes N. When we study the behavior of $\Lambda(b)$ as a function of the critical parameter b, for fixed correlation exponent α, the curves with different system sizes N will cross at a single point. This point indicates the value of the critical parameter $b_c=b_c(\alpha)$ for the transition from extended states to localized states [11].

Besides the methods mentioned above, using relation (3), we will calculate the spatial dependence of the electric current $I(b,\alpha)$ at a given frequency ω, as a function of the parameters b and α, for fixed system size N. In this way we can obtain a complementary information of the localization properties of each state.

3. Numerical results

For the numerical calculation we consider a fixed frequency $\omega=3.6$, and we use arbitrary values of the conductance and inductance: $C_0=0.5$, $L_0=1.0$. In addition, the average of every quantity is calculated using 10^4 samples. To prevent a negative value of capacitances C_j and inductances L_j, the amplitude of the fluctuation b must be less than the minimum between $C_0=0.5$ and $L_0=1.0$ respectively. In this case we have the following condition: $0\leq b<C_0=0.5$.

In first place, we study the behavior of the averaged normalized localization length $\Lambda(b,\alpha)$ as a function of the correlation exponent α and as a function of the fluctuation parameter b, for fixed system size $N=2^{17}$. Fig. 1 shows the map of the extended states ($\Lambda\geq 1$, filled squares) and the localized states ($\Lambda<1$, empty squares). The range of variation of the parameters α and b in this map is the following: $b\in[0.05,0.45]$ and $\alpha\in[0.6,2.4]$. In the map, we can see that for $b=0.05$ the first extended state appears for a minimum α value $\alpha_{\min}=1.36$, and for $b=0.4$ the first extended state appears for $\alpha_{\min}=1.74$. However, for $b=0.45$, it is not possible to find a minimum α value, and all states are localized ($\Lambda<1$, empty squares). For fixed system size $N=2^{17}$, this map indicates the minimum value of the correlation exponent $\alpha_{\min}$ for the appearing of extended states for each b value. It is very important to note that the map of Fig. 1 indicates the behavior only for finite system size. As a consequence, in the thermodynamic limit ($N\to\infty$), the phase diagram separating the localized states from extended states, must be different.

In Fig. 2 we show the behavior of (a) the normalized localization length $\Lambda(\alpha)$, (b) the participation number $D(\alpha)$, (c) the Shannon entropy $S(\alpha)$ and (d) the normalized participation ratio ($N\times IPR$), as a function of the correlation exponent α, for fixed system size $N=2^{17}$, for different values of the fluctuation amplitude b, namely, $b=\{0.05,0.15,0.25,0.35,0.40,0.45\}$. In Fig. 2(a)

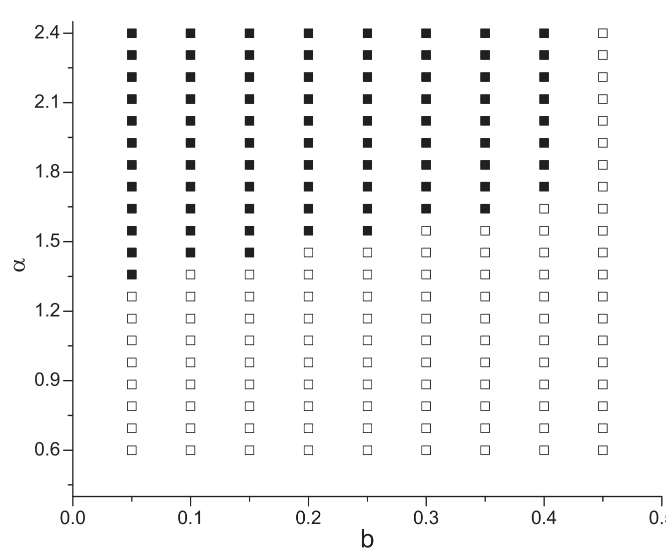

Fig. 1. Map of the extended states ($\Lambda\geq 1$, filled squares) and the localized states ($\Lambda<1$, empty squares) for finite $N=2^{17}$ and $\omega=3.6$, when the parameters α and b varies in the range: $b\in[0.05,0.45]$ and $\alpha\in[0.6,2.4]$.

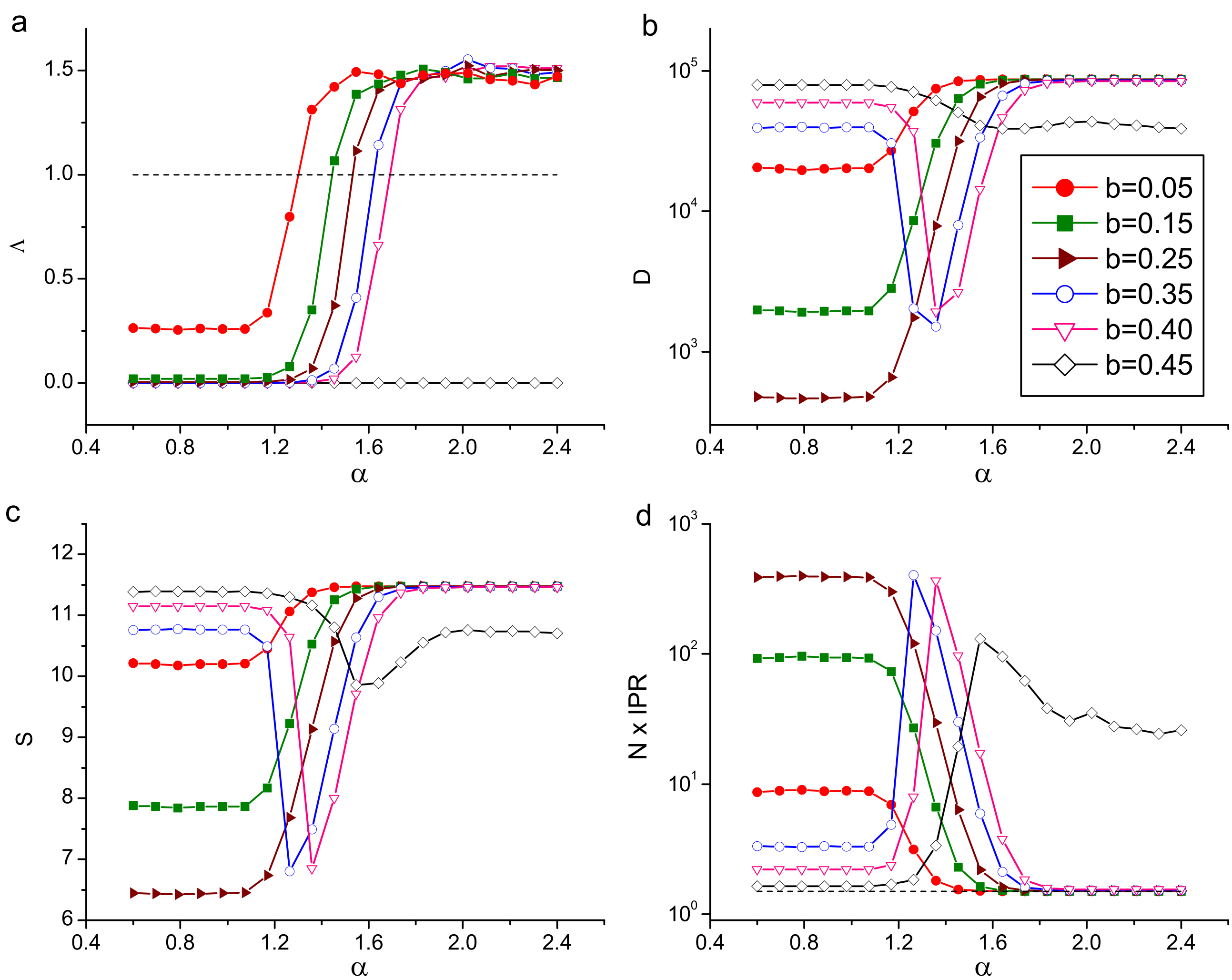

Fig. 2. Behavior of (a) the normalized localization length $\Lambda(\alpha)$, (b) the participation number $D(\alpha)$, (c) the Shannon entropy $S(\alpha)$ and (d) the normalized participation ratio $(N\times IPR)(\alpha)$, as a function of the correlation exponent α, for $N=2^{17}$, for different values of the fluctuation amplitude b. For $b=0.45$ we cannot find extended states. In addition, for $b>0.25$ the functions D, S and $(N\times IPR)$ change their qualitative behavior.

we can see the appearing of extended states ($\Lambda\geq 1$) for all b values studied, starting from a minimum value of the correlation exponent α_{min}, except for the case $b=0.45$ where $\Lambda\approx 0$ for all α values. This result is coincident with the general result shown in the map of Fig. 1. In Fig. 2(b)–(d) we show the behavior of D, S and $(N\times IPR)$, respectively. There we can see basically the same behavior of Fig. 2(a), namely, the appearing of extended states for all b values studied, from a minimum value of the correlation exponent α_{min}, except for the case $b=0.45$, where each quantity shows a behavior very different from the other curves, indicating a localized behavior. In these three pictures, the extended states appear when D, S and $(N\times IPR)$ are very near to the values of the corresponding periodic case. In addition, in the last three pictures we can see a qualitative change of behavior of the quantities D, S and $(N\times IPR)$ for the case $b\leq 0.25$ compared to its behavior for the case $b>0.25$. We emphasize that these results are valid only for finite system size $N=2^{17}$. As a consequence, we can observe a transition from localized states to extended states as a function of the parameters b and α, at least for finite system size $N=2^{17}$ and fixed frequency $\omega=3.6$.

To obtain the behavior of the TL with fluctuating long-range correlated disorder in the thermodynamic limit, we use the participation number $D(b,\alpha,N,\omega)$ to study: (a) the linear relationship between $\ln(D)$ versus $\ln(N)$, (b) the relative fluctuation $\eta_D(b)$ of the participation number D, and (c) the Binder cumulant $B_D(b)$ of the participation number D, as a function of correlation exponent α for different values of b in the range $b\in[0.05,0.48]$.

Case (a). Fig. 3 shows $\ln(D)$ versus $\ln(N)$ for N ranging from $N=2^{10}$ to $N=2^{20}$, for fixed $b=0.25$ and different values of α. We find a linear relationship for $\alpha=1.4$, where the slope m is less than $m=1.0$. For increasing α values, the slope m of the straight line increases, and finally for $\alpha=\alpha_c=1.51$, the slope is exactly $m=1.0$. This critical value is indicated by the linear fit to the numerical data. In this case, the participation number $D(b)$ increases linearly with the system size N, which is an indication of the extended behavior of the electric current function in the thermodynamic limit. For $\alpha\geq\alpha_c=1.51$ the value of every slope is $m=1.0$. Also, we have studied $\ln(\xi)$ and $\ln(S)$ versus $\ln(N)$ to compare the behavior of these quantities in the localized and extended states. Fig. 4(a) shows $\ln(\xi)$ versus $\ln(N)$. This picture shows a behavior very similar to the behavior of $\ln(D)$ versus $\ln(N)$ showed in Fig. 3. However, for the relation $\ln(S)$ versus $\ln(N)$ (see Fig. 4(b)) we cannot find a linear relationship, at least for the range of values of N studied.

Case (b). In Fig. 5 we show the behavior of the relative fluctuation $\eta_D(\alpha)$ of the participation number D, as a function of α for $b=0.25$, for four different values of the system size N, namely, $N=\{2^{13},2^{14},2^{15},2^{16}\}$. In this figure, we can clearly see

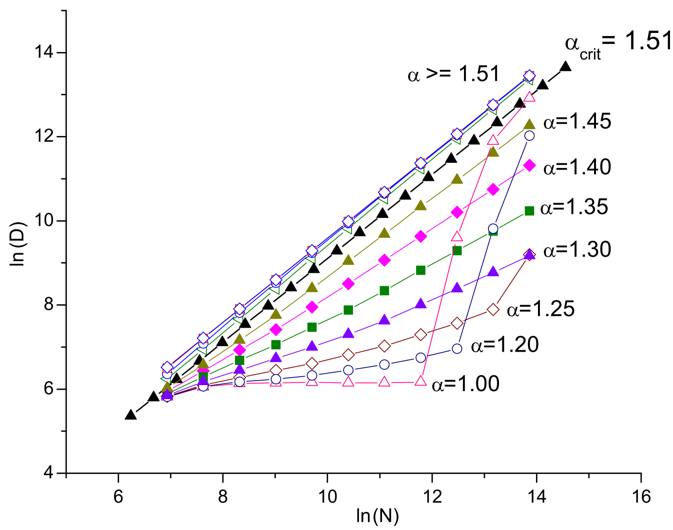

Fig. 3. Study of the behavior of $\ln(D)$ versus $\ln(N)$, for fixed $b = 0.25$, for different values of α. When α increases, the curves tend to straight lines. For $\alpha \geq 1.4$ the slopes m of the straight lines are lesser than 1.0. However, for $\alpha \geq 1.51$ the linear relationship $\ln(D)$ versus $\ln(N)$ has slope $m = 1.0$, indicating the transition to an extended behavior. For the critical value $\alpha_c = 1.51$, we show the straight line obtained by linear fit.

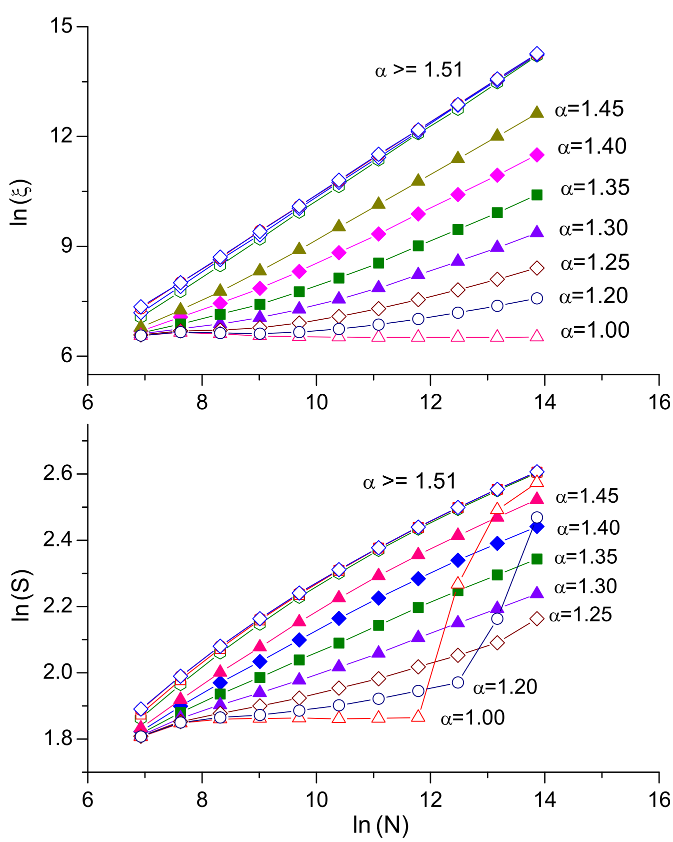

Fig. 4. Study of the behavior of (a) $\ln(\xi)$ and (b) $\ln(S)$, as a function of $\ln(N)$ for fixed $b = 0.25$, for different values of α. (a) When α increases, the relation $\ln(\xi)$ versus $\ln(N)$ behaves in similar way to the behavior of $\ln(D)$ versus $\ln(N)$ indicated in Fig. 3. (b) For the Shannon entropy S, we cannot find a linear relationship between $\ln(S)$ versus $\ln(N)$, at least for the range of values of N studied.

that the critical point appears for $\alpha_c = 1.51$. This critical value is the same critical value found in Fig. 3 studying the linear relationship between $\ln(D)$ versus $\ln(N)$.

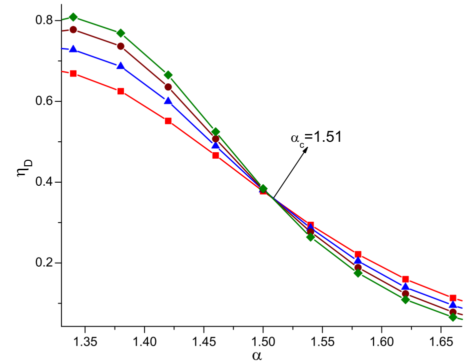

Fig. 5. Relative fluctuation $\eta_D(\alpha)$ of the participation number D as a function of α for $b = 0.25$, for four different values of the system size N, namely, $N = \{2^{13}, 2^{14}, 2^{15}, 2^{16}\}$. The critical point which separates localized states from extended states in the thermodynamic limit appears for $\alpha_c = 1.51$. This values is exactly the same critical value shown in Fig. 3.

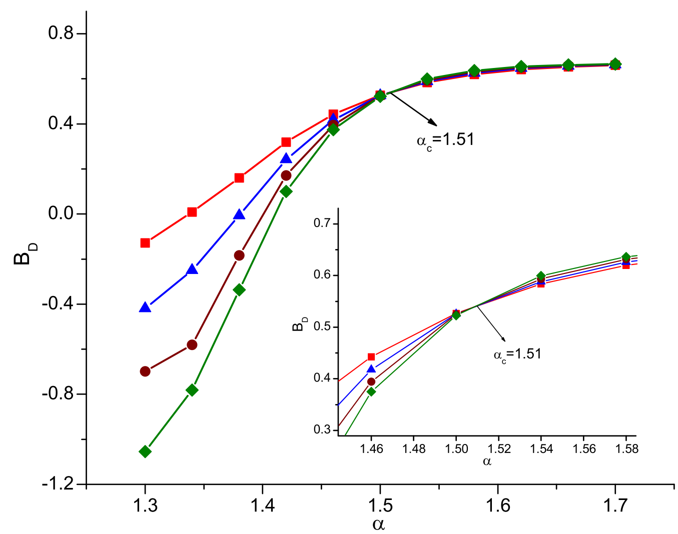

Fig. 6. Binder cumulant $B_D(\alpha)$ as a function of α for $b = 0.25$, for four different values of the system size N, namely, $N = \{2^{13}, 2^{14}, 2^{15}, 2^{16}\}$. The critical point appears for $\alpha_c = 1.51$. This result is coincident with the critical value $\alpha_c = 1.51$ obtained in Figs. 3 and 5.

Case (c). Fig. 6 shows the behavior of the Binder cumulant $B_D(\alpha)$ of the participation number D, as a function of α for $b = 0.25$, for four different values of the system size N, namely, $N = \{2^{13}, 2^{14}, 2^{15}, 2^{16}\}$. In this case we again find exactly the same critical point $\alpha_c = 1.51$. The inset shows the details of the crossing point. This result is totally coincident with the critical value $\alpha_c = 1.51$ obtained in Fig. 3 of the case (a), studying the linear relationship between $\ln(D)$ versus $\ln(N)$ and the critical value $\alpha_c = 1.51$ obtained in Fig. 5 of the case (b), studying the relative fluctuation $\eta_D(b, \alpha, N)$. In Figs. 5 and 6 the average was taken over 5×10^4 configurations to obtain more precise results. In addition, in Fig. 7 we show the spatial dependence of the electric current function $I(b, \alpha)$ at a given frequency $\omega = 3.6$, for $b = 0.25$ and $N = 2^{17}$, for four values of the correlation exponent α in the vicinity of the critical value $\alpha_c = 1.51$. There we can clearly see that $I(b, \alpha)$ is a localized function for $\alpha = 1.30$ and $\alpha = 1.40$, but is an extended function for $\alpha \geq 1.51$ This result is totally coincident with the results obtained by the other methods for the fixed value $b = 0.25$.

Using the procedures describe above, we have studied the linear relationship between $\ln(D)$ versus $\ln(N)$, the relative fluctuation η_D, the Binder cumulant B_D and the spatial dependence of the electric current $I(\alpha, b)$, as a function of α for different b values

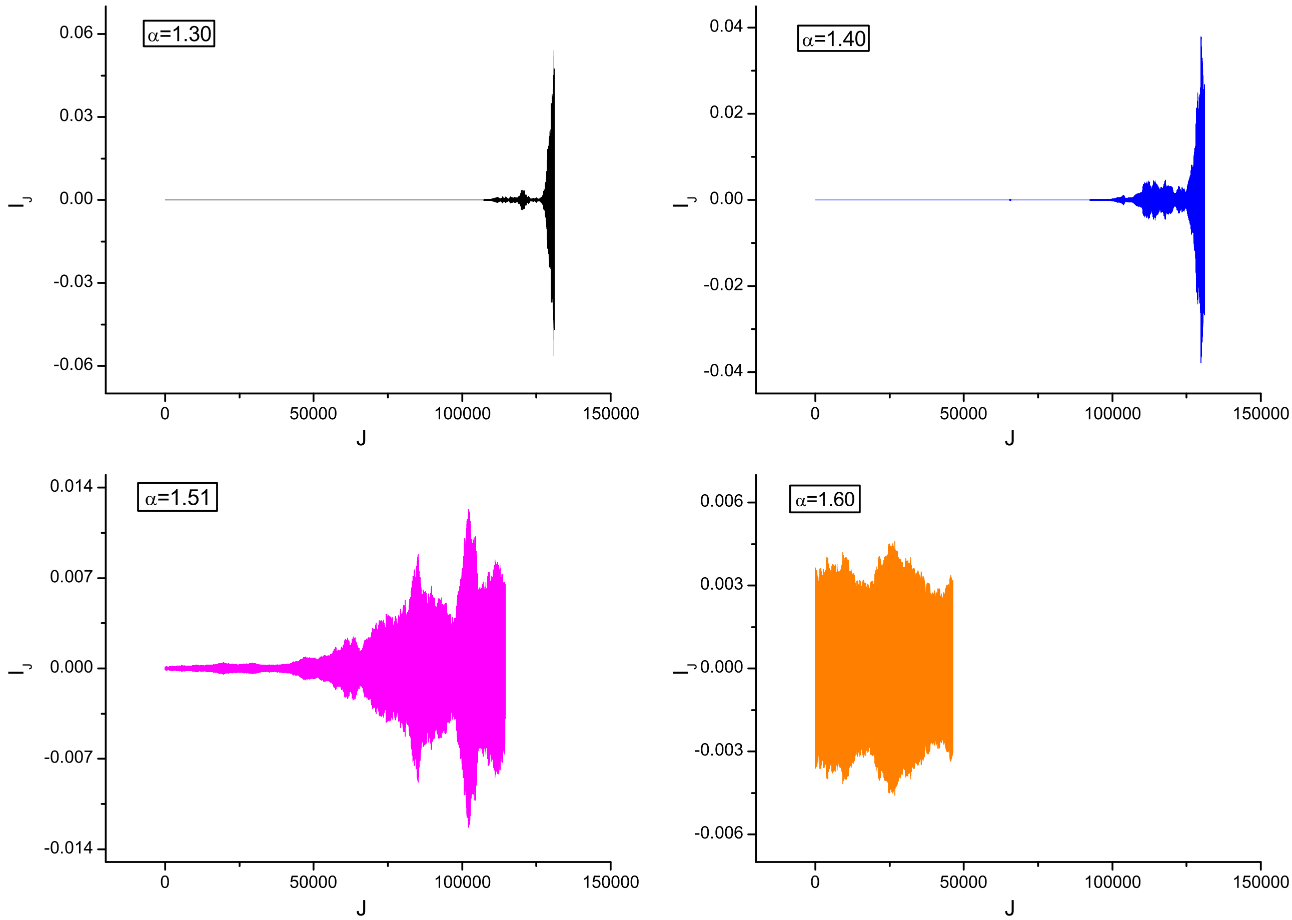

Fig. 7. Electric current function I_j for $b=0.25$, $N=2^{17}$ and $\omega=3.6$. For $\alpha=1.30$ and $\alpha=1.40$, the electric current function is localized, but for $\alpha \geq 1.51$, I_j shows an extended behavior. These results are totally coincident with the results shown in Figs. 3, 5 and 6.

in the range $b \in [0.05,0.48]$. The general results are the following. In first place, for every $b \in [0.05,0.43]$, the critical values $\alpha_c = \alpha_c(b)$ obtained by the three different methods are totally coincident between them, and furthermore, the critical values $\alpha_c(b)$ in this range are practically the same: $\alpha_c = 1.51$, except for the points $b=0.41$, $b=0.42$ and $b=0.43$, where we have $\alpha_c(0.41) = 1.52$, $\alpha_c(0.42) = 1.53$ and $\alpha_c(0.43) = 1.55$. In the second place, for $b > 0.43$, it is not possible to find critical values. The results previously shown in Figs. 1 and 2, where all states are localized states for $b=0.45$, agree with this general result.

As a consequence, to study the localization properties of the electric current function $I(b,\alpha)$ around $b=0.43$, we need to use another method, different from the methods previously used. In this case we study the behavior of the normalized localization length $\Lambda(b)$ as a function of b and as a function of the system size N, for different values of the correlation exponent α. Fig. 8(a) shows the behavior of $\Lambda(b)$ as a function of b, for fixed system size $N=2^{17}$ and different α values, namely, $\alpha = \{1.47, 1.51, 1.55, 1.65, 1.75\}$. This picture shows that $\Lambda(b)$ goes to zero for $b > 0.43$ (approximately), indicating a localized behavior for every α value in this region. Also, for $b < 0.43$ the values of $\Lambda(b)$ grow with increasing α indicating an increase in the localization length $\xi(b)$. Fig. 8(b) and (c) shows $\Lambda(b)$ as a function of the system size N, from $N=2^{13}$ to $N=2^{16}$, for two different values of α, namely, $\alpha = \{1.47, 1.65\}$. For $\alpha = 1.47$ (Fig. 8(b)) the curves do not intersect and as a consequence, there is no critical point separating localized states from extended states. This result is coincident with the result obtained above, because for $\alpha \leq \alpha_c(b) = 1.51$, the electric current function $I(b,\alpha)$ is a localized function. On the contrary, for $\alpha = 1.65$ (Fig. 8(c)) all the curves intersect at the critical point $b_c = 0.43$, which separates extended states ($b \leq b_c = 0.43$) from localized states ($b > b_c$). Fig. 8(d) shows a detail of the intersection point. Using this method for every $\alpha \geq \alpha_c = 1.51$, we have obtained the same critical point $b_c = 0.43$. As a consequence, for $\alpha \geq \alpha_c = 1.51$, we obtain the critical line $b = b_c(\alpha) = 0.43$. The curves of Fig. 8 were obtained using 5×10^4 configurations to obtain more precise results.

Using the critical lines, $b = b_c = 0.43$ and $\alpha = \alpha_c = 1.51$, we build the phase diagram of the transition from localized states to extended states in the plane (b,α) (see Fig. 9). In addition, this result implies that the critical parameter α which characterizes the correlation and the amplitude of disorder b, are completely independent.

Interestingly, a very similar phase diagram (W,p) was found by Shima et al. [11] studying the localization properties of electron eigenstates in one-dimensional systems with long-range correlated diagonal disorder. The parameter p is related with the correlation exponent α of the Fourier filtering method through the relation $p = (2\alpha - 1)$, and W is the distribution width defined by the relation $\varepsilon_j \in [-W/2, W/2]$, which characterizes the diagonal disorders through the site energies ε_j. In our work, we have diagonal and off-diagonal disorder through the capacitances $C_j \in [C_0 - b, C_0 + b]$ and inductances $L_j \in [L_0 - b, L_0 + b]$. In Ref. [11] one of the critical lines of the phase diagram is $p=2.0$. This critical line is practically the same critical line of our work, located at $\alpha = 1.51$, which corresponds to $p=2.0$. In addition, for the amplitude b of the fluctuation of C_j and L_j, we find a critical line

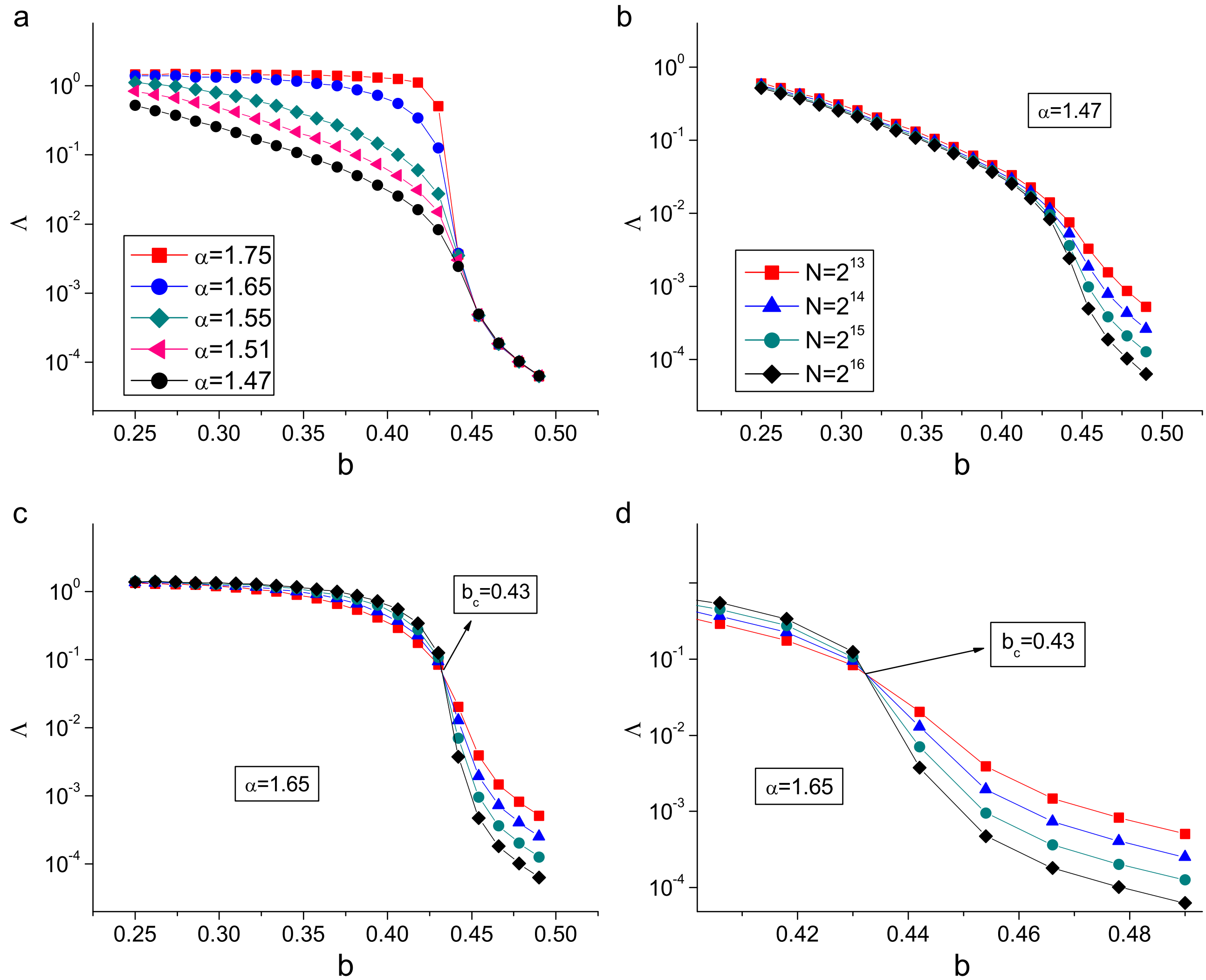

Fig. 8. Study of the normalized localization length $\Lambda(b)$. (a) $\Lambda(b)$ for five different values of α for fixed $N=2^{17}$. For $b>0.43$ all states are localized because $\Lambda(b)\rightarrow 0$, but for $b\leq 0.43$ the localization length $\xi(b)$ increases with increasing α values. For pictures from (b) to (d), N varies from $N=2^{13}$ to $N=2^{16}$. (b) $\Lambda(b)$ for $\alpha=1.47$ (there is no a critical point). (c) $\Lambda(b)$ for $\alpha=1.65$ (a critical point appears for $b_c=0.43$). (d) Detail of the critical point $b_c=0.43$ for the case $\alpha=1.65$.

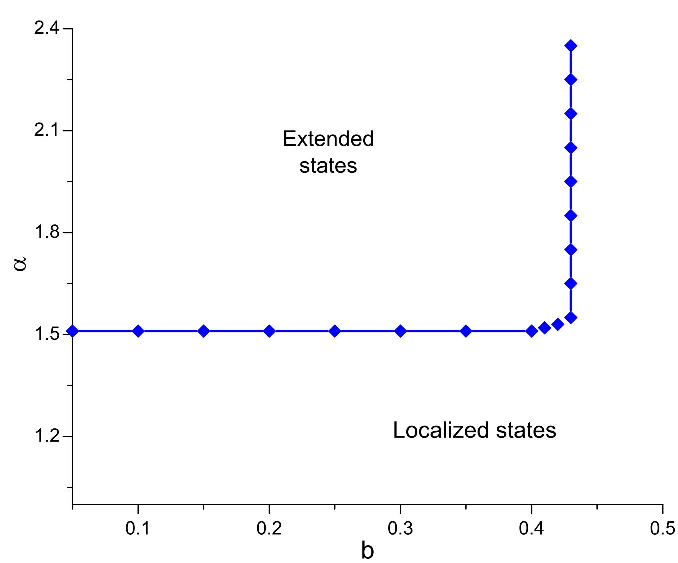

Fig. 9. Phase diagram (b,α) separating localized from extended states in the thermodynamic limit. This phase diagram is formed by two critical independent straight lines: $b=b_c=0.43$ and $\alpha=\alpha_c=1.51$.

$b=b_c=0.43$, which is reminiscent of the critical line founded in Ref. [11] $W=W_c=4.0$.

4. Secure communication

In this section we study the effect produced in the localization properties of the transmission line when we change the values of capacitances and or inductances in one or more sites of the TL with random values R, with $R\in(0,1)$. This procedure introduces alterations in the original sequence given by the relationship (2), which determines the distribution of capacitances $\{C_k\}$ and inductances $\{L_k\}$. We can introduce random values in four different cases: (a) we change only the capacitances, $C_n=R$, (b) we change only the inductances, $L_n=R$, (c) we change both quantities with different random values R_1 and R_2, namely, $C_n=R_1$ and $L_n=R_2$, with $R_1\neq R_2$, and (d) the change is the same in both quantities, namely, $C_n=L_n=R$. It is important to note that in the average procedure, the sites to be altered are chosen at random.

To numerically show the profound changes that occur in the localization properties of the TL, we study the normalized localization length $\Lambda(\alpha)$ as a function of the correlation exponent α, for fluctuation amplitude b=0.25, fixed frequency $\omega=3.6$ and

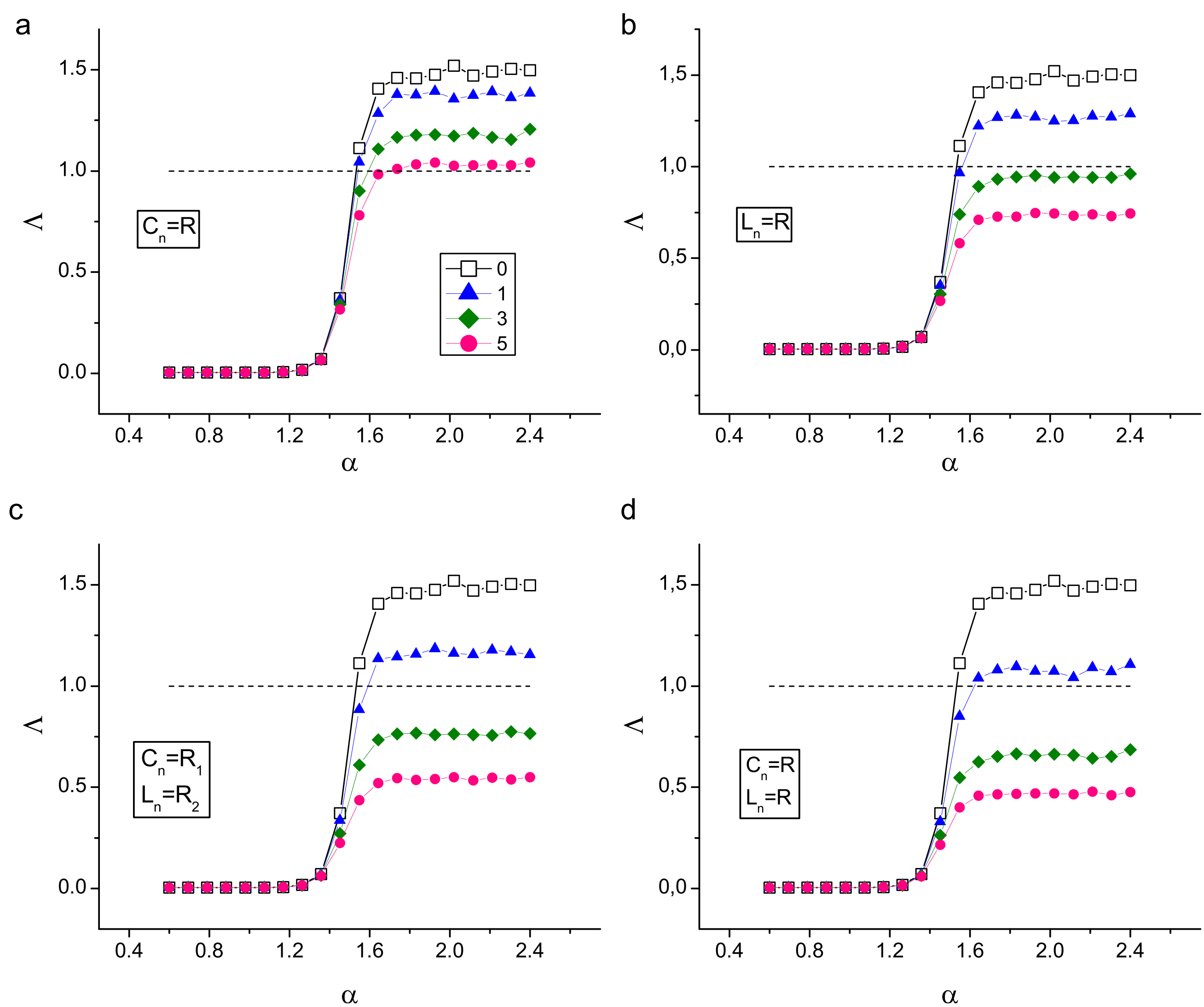

Fig. 10. Normalized localization length $\Lambda(\alpha)$ as a function of α for $b=0.25$. We show four different forms in which we can introduce 1, 3 or 5 impurities, alternating in this way the original long-range correlated sequence (see Section 4). The most important effect in the normalized localization length occurs for the case $C_n=L_n=R$ (Fig. 10(d)), namely, when the impurities are introduced in phase, in one or more sites.

fixed system size $N=2^{17}$. For this specific case, $b=0.25$, we have found a phase transition from localized states to extended states for the critical value $\alpha_c=1.51$ (see Figs. 3, 5 and 6). Fig. 10 shows $\Lambda(\alpha)$ as a function of α for the four cases indicated above. In every case we compare the value of the original sequence (open square) with the sequence where 1, 3 or 5 sites are changed simultaneously (filled symbols). There we can see that the main effect of the alteration of the original long-range correlated sequences, is the decrease in the values of $\Lambda(\alpha)$ for the four cases studied. This means that the TL jumps from extended states to localized states for $\alpha \geq \alpha_c = 1.51$, except for the case $C_n=R$ (see Fig. 10(a)), where we must change more than 5 sites to obtain localized states. On the contrary, the case $C_n=L_n=R$, shows the most pronounced effect (see Fig. 10(d)), because even for the case of 1 random change, we have $\Lambda(\alpha) \gtrapprox 1$ and for 3 random changes, the TL goes to localized states ($\Lambda(\alpha)<1$).

This general behavior can be considered as an interesting effect that can be used in secure communication [20,19], because any attempt of external connection to the transmission line modifies the long-range correlation sequence and therefore will destroy the band of extended states closing the transmission and stopping the communication. When we talk about secure communication, we want to stress that a device using a transmission line accordingly to our long-range correlation sequence only will allow the communication if there are no external connections and therefore it is intrinsically secure. In addition, in the thermodynamic limit the phase diagram showed in Fig. 9 disappears, because the localized states only survive. The changes in the localization properties are very pronounced for the case $C_n=L_n=R$ (R random number), because any little alteration in the original sequence changes the localization length abruptly. As a consequence, the transmission line jumps abruptly from an extended state to a localized state. This effect is more pronounced when α is very near to the transition point α_c in the plane (α,Λ) (see Fig. 10(d)). In applications, it is necessary that a long sequence with $N>2^{15}$ really prevent any intrusion in the communication.

5. Conclusions

In conclusion, we have studied the localization properties of the classical dual transmission lines when we introduce long-range correlated disorder in the distribution of capacitances and inductances. The specific disorder is given by the following relations: $C_j = C_0 + b\sin(2\pi x_j)$, $L_j = L_0 + b\sin(2\pi x_j)$, where $\{x_j\}$ is obtained from de Fourier filtering method. To study the localization behavior of the disordered dual transmission lines, we consider a set of quantities which depend of the frequency ω, the fluctuation amplitude b, the correlation exponent α, and the system size N. We calculate the electric current function $I(\omega,b,\alpha,N)$, the localization length $\xi(\omega,b,\alpha,N)$, the normalized localization length $\Lambda(\omega,b,\alpha,N)$, the Shannon entropy $S(\omega,b,\alpha,N)$, the participation number $D(\omega,b,\alpha,N)$ and the inverse participation ratio $IPR(\omega,b,\alpha,N)$. We use three different methods to obtain the critical behavior of the participation number $D(\omega,b,\alpha,N)$: (a) study of the linear relationship between $\ln(D)$ and $\ln(N)$, (b) study of the relative fluctuation $\eta_D(\omega,b,\alpha,N)$ as a function of the system size N and (c) study of the Binder cumulant $B_D(\omega,b,\alpha,N)$ as a function of the system size N. These three different methods give exactly the same critical value $\alpha_c(b)$ for each b value. In addition, to obtain the critical curve $b_c(\alpha)$, we study the behavior of the normalized localization length $\Lambda(\alpha,N)$ as a function of the system size N for different values of the correlation exponent α. With these results we build the phase diagram in the thermodynamic limit, which separates the localized states from the extended states. Another interesting result is the disappearance of the conduction bands when we introduce a finite number of impurities in random sites. This process can serve as a mechanism of secure communication, because any attempt of external connection to the transmission line modifies the long-range correlation sequence and therefore will destroy the band of extended states closing the transmission and stopping the communication.

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